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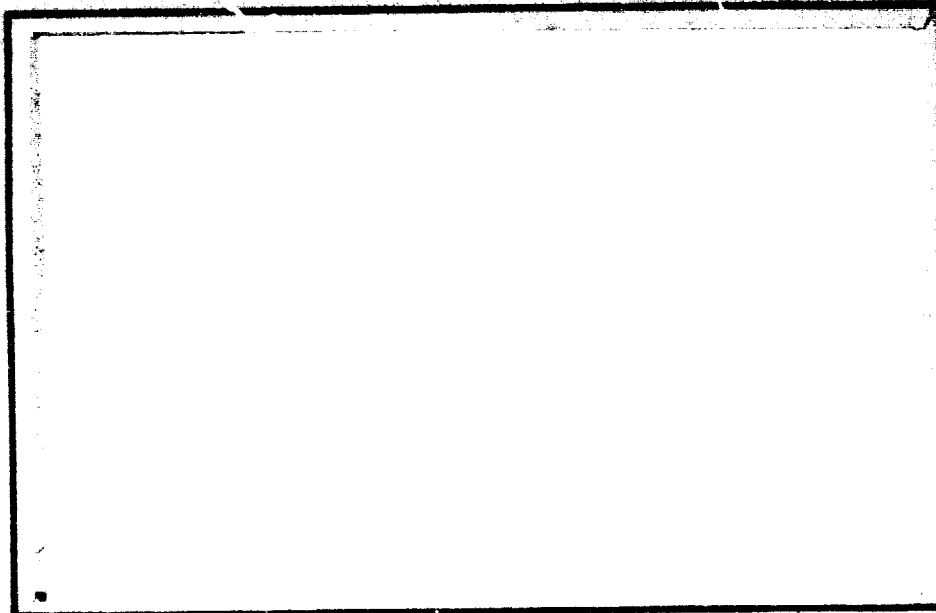
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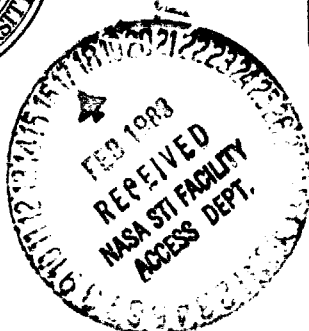
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# COLLEGE of ENGINEERING



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**BLACKSBURG,  
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UNSTEADY BOUNDARY-LAYER INJECTION

by

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August 1981  
VPI-E-81-23

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### Abstract

The boundary-layer equations for two-dimensional incompressible flow are integrated numerically for the flow over a flat plate and a Howarth body. Injection is introduced either impulsively or periodically along a narrow strip. Results indicate that injection perpendicular to the wall is transmitted instantly across the boundary layer and has rather little effect on the velocity profile parallel to the wall. The effect is a little more noticeable for flows with adverse pressure gradients. Injection parallel to the wall results in fuller velocity profiles. Parallel and oscillatory injection appears to influence the mean. The amplitude of oscillation decreases with distance from the injection strip but further downstream it increases again in a manner reminiscent of an unstable process.

## Introduction

Blowing and suction has been an effective method of boundary-layer control for many years. One of the important problems, namely how blowing and suction specifically affect the stability of a boundary layer has been addressed extensively. In this short report the effects of unsteady injection are examined. However, this study concentrates on the response of a laminar boundary layer to unsteady vectored injection. The interpretation of the results and their significance in the study of flow stability are not discussed here. Some qualitative conclusions based on the fullness of the velocity profile can perhaps be drawn, but the issue of stability is not even addressed here.

The purpose of this pilot project was to demonstrate the feasibility of the idea of stability control by injection. To this end the laminar boundary-layer equations were integrated numerically. During the short period of this investigation it became obvious that this is a rather limited approach to the problem.

Unsteady injection perpendicular and parallel to the wall was tested numerically. The disturbance was introduced impulsively in the flow over a flat plate. The effect of impulsively started injection for the flow over a body with adverse pressure gradient was also examined. Finally, oscillating injection parallel to the wall was calculated. This report also contains suggestions on how to attack the problem in a more effective way through the boundary layer equations, as well as some comments on how the real flow will respond and what to anticipate if the full Navier-Stokes equations are employed. Because of the limitations of time for this project as well as the limitation of this short report, a lot of information about the actual numerical work has been left

out. However, the method employed has been described adequately in earlier publications which are referenced here. This report should therefore be considered a continuation of earlier published material.

# 1. Numerical Results

Consider the unsteady boundary-layer equations in their dimensionless form

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad (1.1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} \quad (1.2)$$

Note that  $v$  and  $y$  are already stretched by the square root of the Reynolds number. Numerical integration is implemented in a transformed plane in terms of Görtler independent variables

$$\xi = \int_0^x U_e dx \quad (1.3)$$

$$\eta = \frac{U_e}{\sqrt{2\xi}} y \quad (1.4)$$

where  $U_e(x,t)$  is the outer edge velocity. New dependent variables are also introduced,

$$F = \frac{u(x,y,t)}{U_e(x,t)} \quad (1.5)$$

$$v = -\frac{2\xi}{U_e} \left( v + \frac{\partial \eta}{\partial x} F \sqrt{2\xi} \right) \quad (1.6)$$

A finite-difference method for solving the system of equations has been developed and adequately described in earlier publications (Telionis, Tsahalis & Werle 1973). It employs an implicit scheme of integration and marches first in the direction of the axial distance  $\xi$ . Then it increments time and repeats the process by sweeping again the



physical space from the leading edge or a stagnation point. More details the reader will find in earlier publications as referenced in Telionis (1979).

In this effort two body configurations were considered; a flat plate and a Howarth body. The second is a fictitious body with an outer flow linearly decelerating according to the formula

$$U_e = 1 - Ax \quad (1.7)$$

It is equivalent to the flow over an ever steepening slope and therefore an increasing adverse pressure gradient.

On the flat plate blowing was introduced in the domain  $0.38 < \xi < 0.59$ , after a dimensionless time  $t = 0.5$  had elapsed. The calculated vertical component at  $\xi = 0.48$  is plotted in Fig. 1.1 for different times. It appears that the entire boundary layer is influenced instantly and the vertical component continues growing with time. Moreover, the effect of blowing becomes stronger away from the wall. In other words, the disturbance actually grows with distance from the wall. For steady flow, this effect has been predicted by triple-deck theory (Nayfeh, Reed and Ragab, 1980). According to this theory the disturbance of the  $v$ -component in the middle deck which is essentially what is shown in Fig. 1.1 should be proportional to the mean profile

$$v = - U(y) \frac{dA(x)}{dx} \quad (1.8)$$

where the function  $A$  is a function of an appropriate axial scale and  $U(y)$  is the Blasius profile. Therefore, the disturbance on  $v$  grows with

distance  $y$  essentially like the  $u$ -component of the basic flow.

The effect of blowing on the  $v$ -component further downstream is still evident as shown in Fig. 1.2, but not as violent as in Fig. 1.1. However, the  $u$ -component of velocity is practically unaffected by blowing. This is clear in Fig. 1.3. There is a reasonable justification for this fact which is discussed in the following section.

The effect of disturbances becomes more evident on boundary layers with adverse pressure gradients, both favorable and unfavorable. In the present study the decelerating flow given by Eq. (1.7) with  $A = 0.12$  was chosen which according to Howarth (1938) separates at approximately  $\xi = 1$ , a fact that was verified by the present method. Separation of course is not of interest here but the sequence of profiles that lead to separation are typical of a boundary layer developing against an adverse pressure gradient. A family of such profiles calculated with our steady flow program is shown in Fig. 1.4 for different  $\xi$ -locations. The fuller profiles correspond to stations closer to the leading edge.

Blowing perpendicular to the wall was introduced again as before at a rate of  $V_w = 0.4$ . The resulting  $u$ -components of the velocity are shown in Figs. 1.5 and 1.6 for times  $t = 0.25$  and  $t = 0.75$  respectively. The effect is again negligible.

Unsteady blowing but parallel to the wall was also introduced at the same stations and on the same flow field, namely the decelerating flow shown in Fig. 1.4. The results are shown in Fig. 1.7 and 1.8 for  $t = 0.25$  and  $t = 0.75$  respectively. Again with  $u_w = 0.05$ , the velocity profile is displaced from the origin, but little qualitative difference can be found.

The method of plotting of Figs. 1.4 - 1.8 is perhaps not the most

appropriate because to compare profiles developing in time requires observation of more than one figure. To bring out clearly the unsteady effect, results were also plotted for a fixed station but at different dimensionless times. Blowing in the interval  $0.48 < \xi < 0.56$  at a rate of  $V_w = 0.2$ , injected impulsively after  $t = 0.25$ , resulted in the sequence of profiles shown in Fig. 1.9. The first three profiles correspond to  $\xi = 0.18, 0.28$  and  $0.38$  and therefore undisturbed flow for all times, because blowing is affected before  $\xi = 0.48$ . A slight change with time in these profiles is due to small adjustments in the solution, as the fictitious stagnation profile is iterated at each time step. The families of curves in 1.9d-h correspond to the undisturbed flow and the first three subsequent time steps with  $\Delta t = 0.25$  respectively. The profiles become less and less full as time progresses. In fact separation is moving upstream and into the domain of integration and as a result the solution does not converge at  $\xi = 0.88$  and  $t = 0.75$ . This is why only three profiles are shown in Fig. 1.9h. The case of suction has also been tested but the results are very similar with the opposite trend and are not shown to save space.

Unsteady impulsive injection parallel to the wall at the same blowing strip and time and with a rate of  $u_w/U_e = 0.05$  results in the profiles of Figs. 1.10a-b. The results for  $u_w/U_e = 0.075$  indicate more clearly the development of the profiles as shown in Fig. 1.11. In these and all subsequent figures the profiles that correspond to stations  $\xi = 0.18, 0.28$  and  $0.38$  are not shown because at these stations the flow is not disturbed and therefore the profiles are identical to those plotted in Fig. 1.9a, b and c.

Finally the effect of oscillatory injection parallel to the wall

was tested with blowing through the same strip, namely  $0.48 < \xi < 0.56$  but with a sinusoidal magnitude,

$$\frac{u_w}{U_e} = B \sin \omega t$$

The dimensionless angular velocity  $\omega$  was set equal to  $2\pi$  and  $B$  was given the values of 0.05 and 0.075. The results for these two values of  $B$  are shown in Figs. 1.12 and 1.13 respectively. The profiles plotted correspond to 16 times equally spaced in one period. They were obtained after three time periods have elapsed to allow for a purely periodic motion to set in. Plotting in this manner results in overlapping of profiles as can be seen in Fig. 1.13d. The results as plotted therefore can serve only as a qualitative guide or perhaps as indicators of the envelope of the oscillating velocity profiles.

It is interesting to note that further downstream from the disturbance strip, the amplitude of oscillation quickly decreases, but as separation is approached, it once more increases sharply. In fact even more interesting is the fact that near separation, the smaller blowing amplitude gives rise to a much more violent oscillation as shown in Fig. 1.12f compared to Fig. 1.13d.

What should be very interesting to investigate here is the mean of the oscillating profiles. Earlier studies (Telionis and Romaniuk, 1978) indicate that periodic oscillations imposed externally induce a steady streaming component on the mean profile. Steady streaming (Riley, 1967) is a net flow always in the direction of the adverse pressure gradient. It could, therefore, induce a fuller mean profile which could be more stable, provided that the frequency of the injected oscillation is

away from the amplification range.

## 2. Discussion

The most characteristic property of impulsive motions in fluids is dwarfing of the convective terms compared to acceleration and diffusion. Blasius (1908) recognized, for example, that for a boundary layer started impulsively, the boundary-layer equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2.1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} \quad (2.2)$$

can be approximated by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2.3)$$

$$\frac{\partial u}{\partial t} = - \frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} \quad (2.4)$$

For classical boundary conditions (no-slip, no-penetration, etc.) the solution is essentially the complementary error function

$$u(x,y,t) = U_e(x) \operatorname{erfc} \frac{y}{\sqrt{2t}} \quad (2.5)$$

where  $U_e(x)$  is the outer flow velocity distribution after the impulse. The idea has been expanded upon first by the classical by now paper of Goldstein and Rosehnead (1938) and then by a large number of subsequent publications (see review articles of Stuart (1971) and Telionis (1979)). The profile of Eq. (2.5) contains the "local effect" in time

which has been anticipated all along in the present analysis, namely for very small times, the effect of the impulsive motion is confined to very small distances away from the wall. The situation is depicted schematically in Fig. 2.1.

If a change is introduced impulsively on an already developed flow, the disturbance is governed again by equations similar to Eqs. (2.1) and (2.2) (Sears, 1949; Telionis, 1981). Moreover, if instead of the outer flow, it is the wall conditions that change impulsively, the character of the solution remains the same and the boundary conditions are readjusted. A classical problem that falls in this category is the Rayleigh problem: an infinite flat plate given impulsively a finite velocity  $u_w$  in a fluid otherwise at rest. Then the boundary conditions are

$$u = 0 \quad \text{at } t < 0 \quad (2.6)$$

$$u = 0 \quad \text{as } y \rightarrow \infty \quad (2.7)$$

$$u = u_w \quad \text{at } y = 0, t > 0 \quad (2.8)$$

and the solution becomes

$$u = u_w \operatorname{erf} \frac{y}{\sqrt{2t}} \quad (2.9)$$

where  $u_w$  is the speed of the wall, as shown schematically in Fig. 2.2.

Unsteady injection parallel to the wall in an established boundary layer can be calculated by virtue of Eqs. (2.3) and (2.4) which govern the disturbance of the flow.

Impulsive injection perpendicular to the wall is a completely different problem. First of all, the counterpart of Rayleigh's problem

does not exist. If an infinite flat plate is given impulsively a velocity perpendicular to its plane rather than parallel to it, a uniform flow is generated instantly, assuming of course that the fluid is incompressible. A schematic of the flow field is shown in Fig. 2.3a. It is due to the viscous effects that in Fig. 2.2 disturbances diffuse away from the wall according to the erf function. A more similar problem to the one under investigation here is perhaps injection through a finite opening on the plate. This then becomes essentially the problem of an impulsively started jet, shown schematically in Fig. 2.3b. Again, at the first instant, the solution should be inviscid and therefore a uniform jet should extend instantly all the way to infinity. Only in finite time the velocity profile readjusts to a jet profile by virtue of viscosity and perhaps may start spreading sidewise.

If flow is injected perpendicular to the wall in an established boundary layer, then it should instantly penetrate across the entire thickness of the layer. This should be the behavior of the actual flow and this appears to be the response of the unsteady boundary-layer equations, as discussed in the previous section. Since inviscid motion in the  $y$ -direction is correctly modeled in the boundary layer equation, this behavior should have been expected. However, for later times the model is not appropriate. The diffusion of the  $v$ -component of velocity, namely the terms  $\partial^2 v / \partial x^2 + \partial^2 v / \partial y^2$  are altogether missing from the boundary layer equations, Eqs. (2.1) and (2.2). This is because the second momentum equation has been eliminated altogether in the boundary-layer approximation. The reader should be reminded that it is due to diffusion terms that the erf or erfc functions appear in the impulsive motions given by Eqs. (2.5) and (2.9). The effect of localized

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disturbances in space and time is therefore not present in the solution of the boundary-layer equations, but it should not be a feature of the real flow either. This is because the establishment of a uniform unsteady jet is a purely inviscid phenomenon, dominated for small times by continuity. At  $t = 0$  it is simply plug flow with speed  $v_w$  throughout and a width equal to the width of the injection opening.

Steady blowing perpendicular to the wall cannot be analyzed via the boundary-layer equations unless the rate of blowing is very small. If small blowing is introduced impulsively, then there is no justification again for neglecting the second momentum equation since the term  $\partial v / \partial t$  may be large, even if  $v$ ,  $\partial u / \partial x$ , etc., are small. A more correct set of equations then should be

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2.10)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} \quad (2.11)$$

$$\frac{\partial v}{\partial t} = - \frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial y^2} \quad (2.12)$$

The disturbance affects the pressure distribution in the vicinity of blowing. Pressure is no longer constant across the boundary layer and should be considered a dependent variable. This set of equations can be integrated numerically by the numerical method employed in the previous section. To this end, a second parabolic equation and a new dependent variable will have to be added to the system. Inviscid boundary conditions can be imposed at the edge of the boundary layer. The  $v$ -component of the velocity will not be matched with its potential flow



counterpart as is usually the practice in uninteracted calculations. A more appropriate procedure is perhaps to consider the feedback from the disturbed potential flow. Allowing the boundary layer to interact with the outer potential flow will make the numerical solution equivalent to a Navier-Stokes solution or a triple-deck asymptotic analysis.

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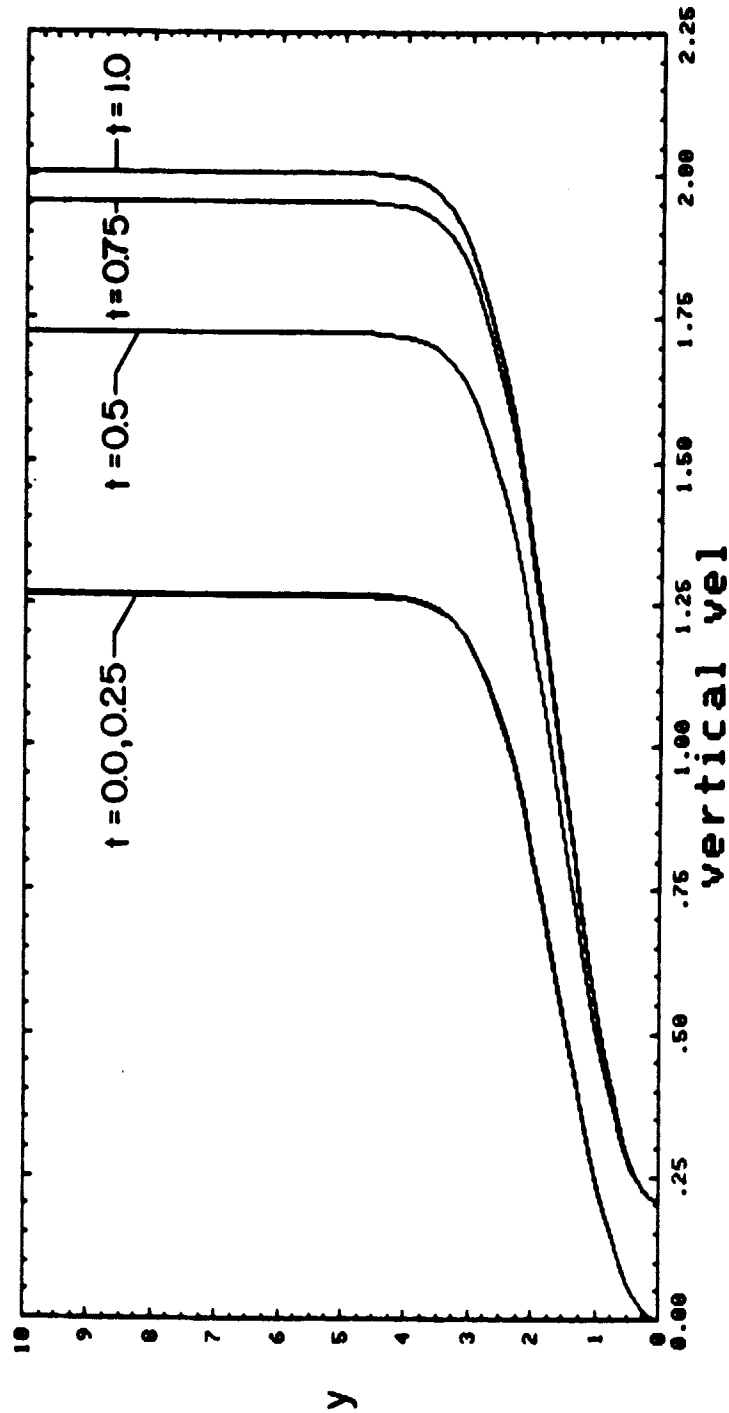


Fig. 1.1 The v-component of velocity at  $\xi = 0.48$  for blowing perpendicular to the flat plate wall.

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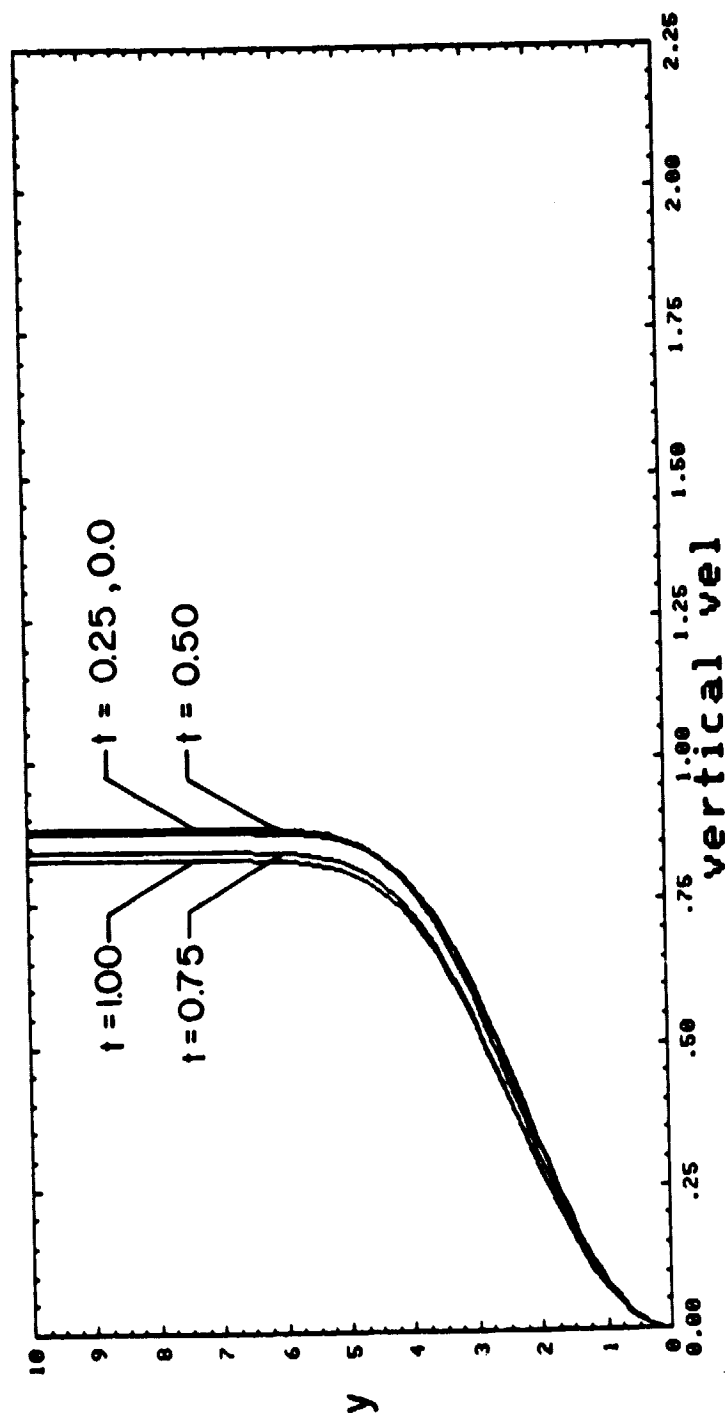


Fig. 1.2 The v-component of velocity at  $\xi = 0.95$  for blowing perpendicular to the flat plate wall.

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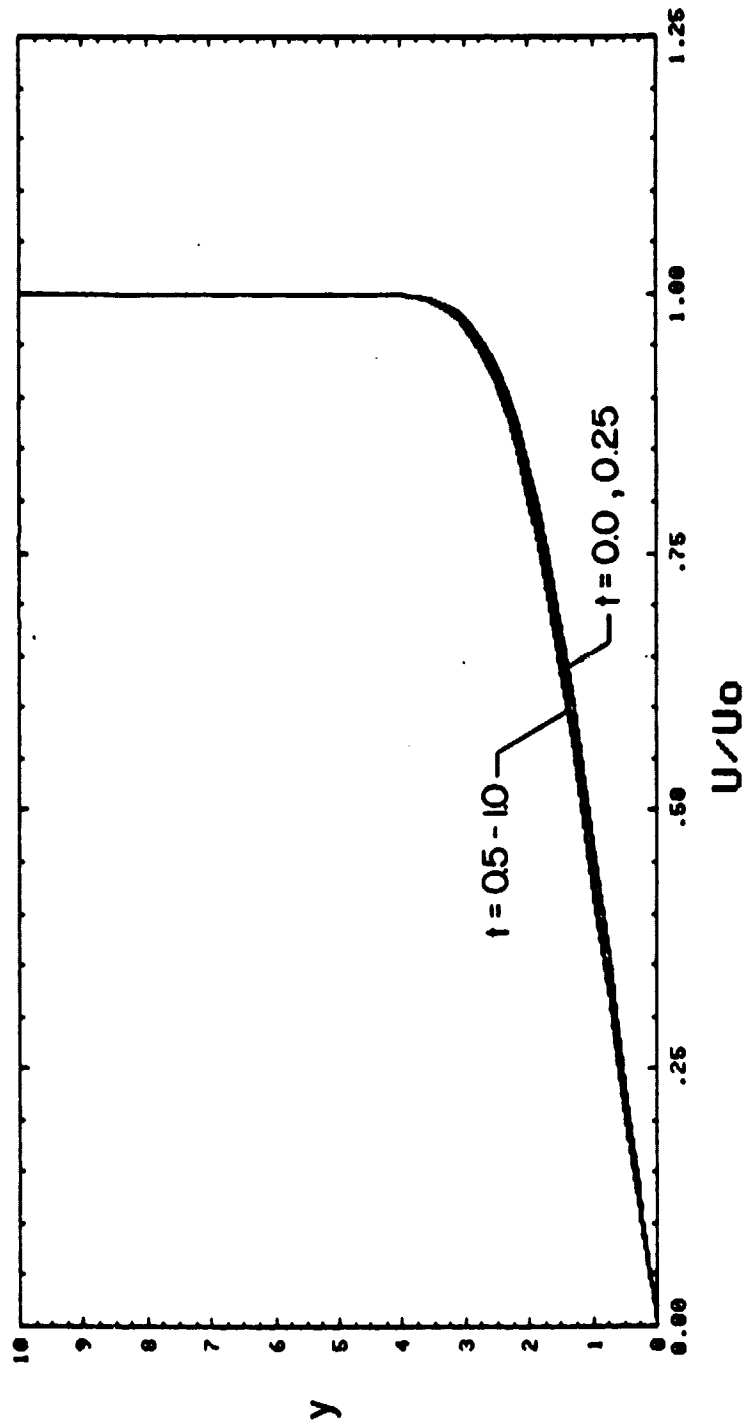


Fig. 1.3 The  $u$ -component of velocity at  $\xi = 0.48$  for blowing perpendicular to the flat plate wall.

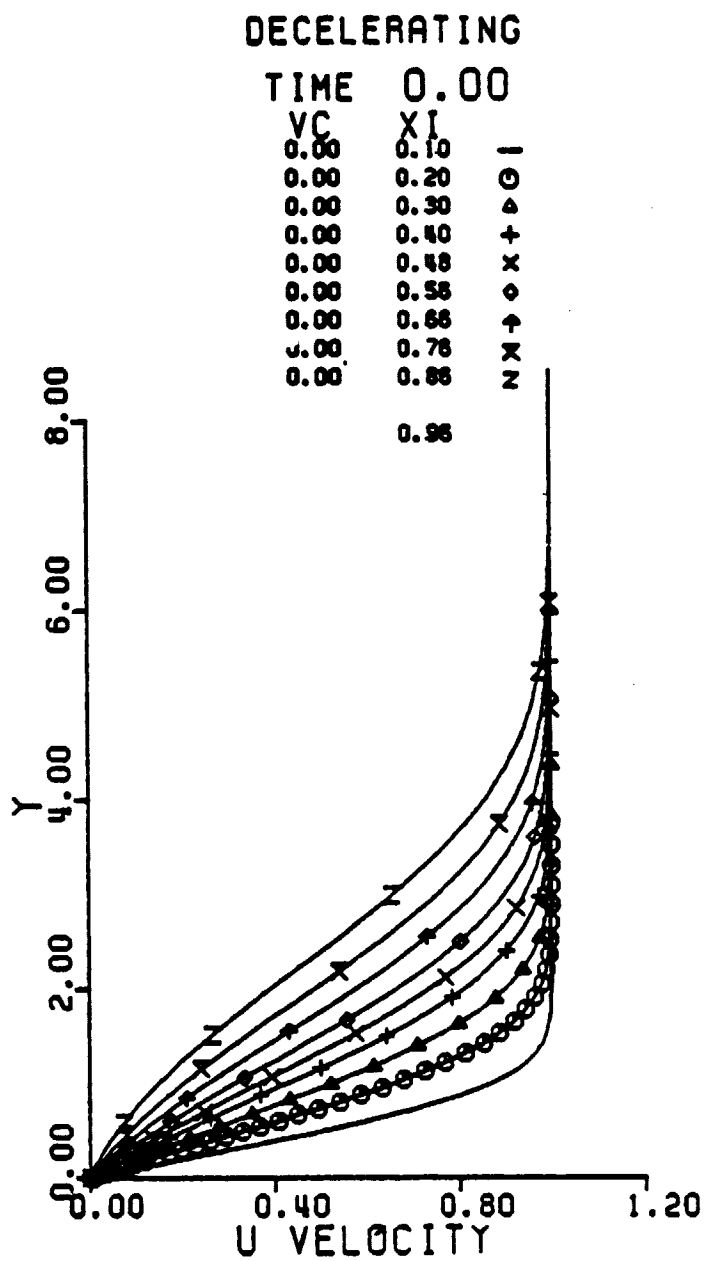


Fig. 1.4 Velocity profiles over the Howarth body given by Eq. 1.7.

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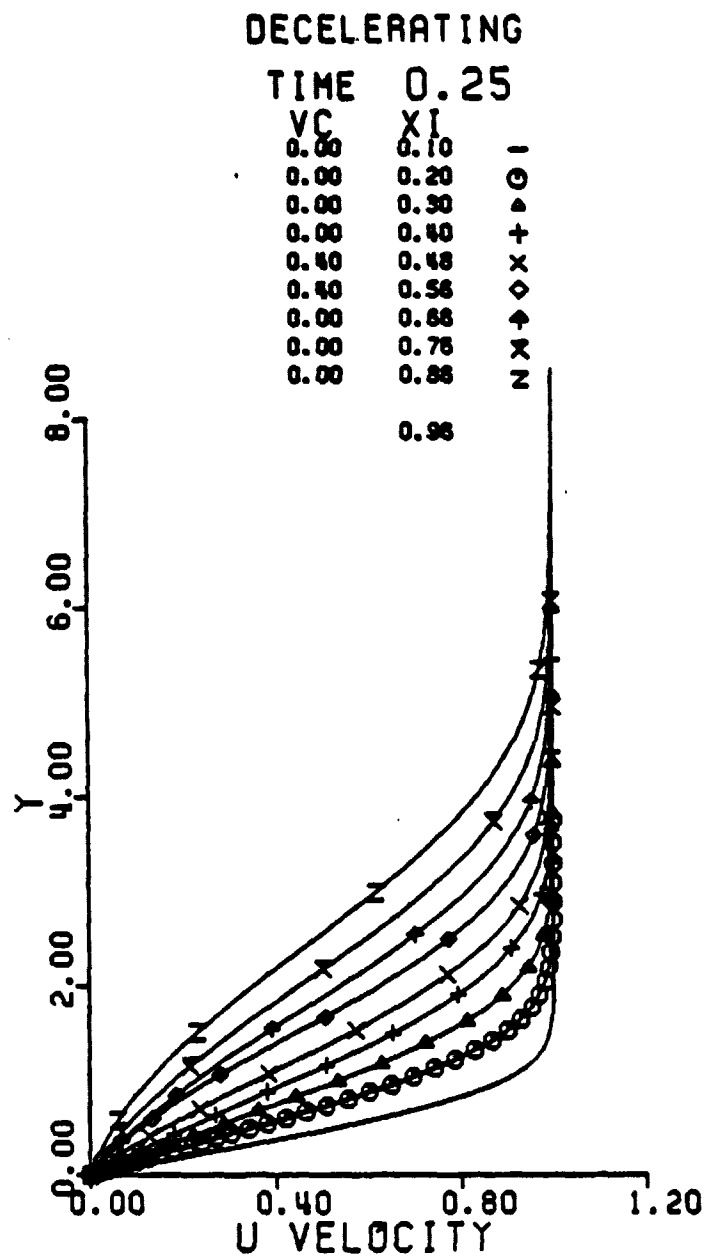


Fig. 1.5 The influence of blowing perpendicular to the wall; Time  $t = 0.25$

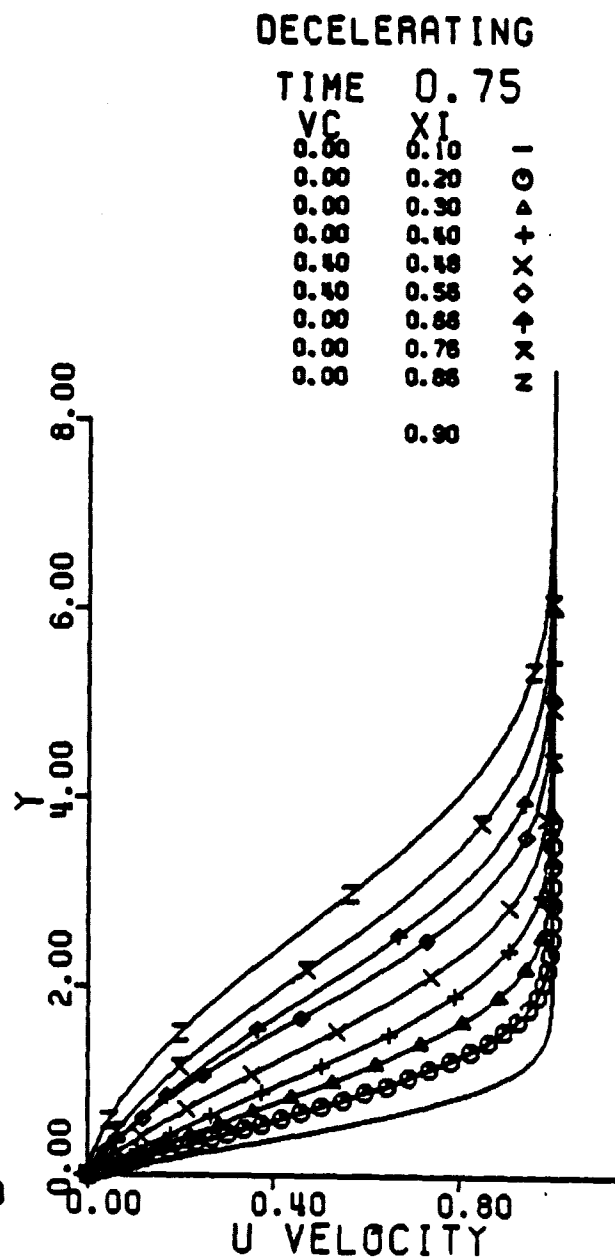


Fig. 1.6 The influence of blowing perpendicular to the wall; Time  $t = 0.75$

DECELERATING

TIME 0.25

UC	XI	
0.00	0.10	○
0.00	0.20	△
0.00	0.30	+
0.00	0.40	x
0.05	0.48	◇
0.05	0.56	↑
0.05	0.66	x
0.05	0.76	z
0.05	0.86	z
	0.96	

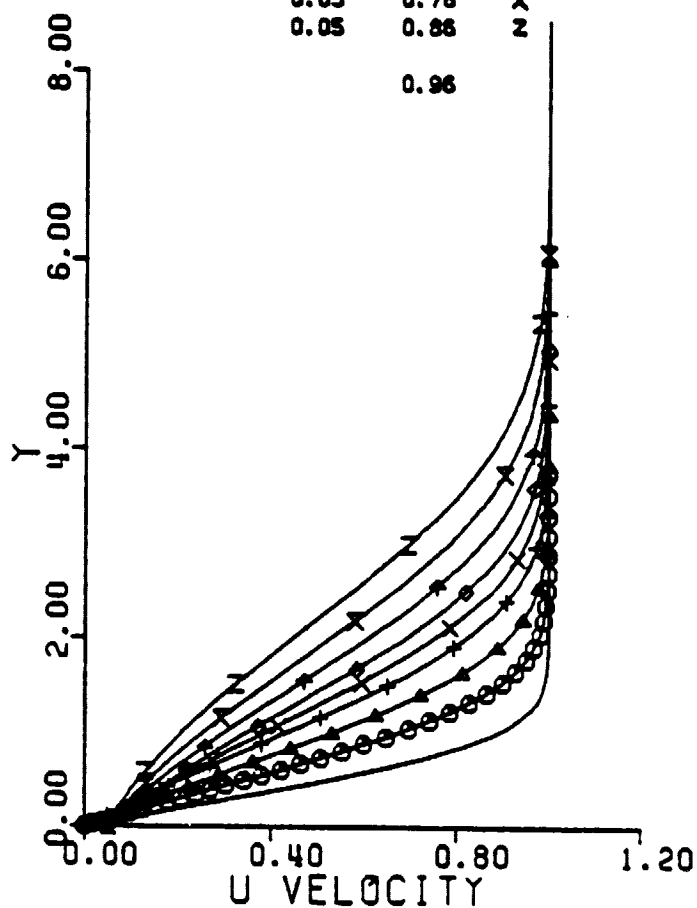


Fig. 1.7 The influence of blowing parallel to the wall;  
Time  $t = 0.25$

DECELERATING

TIME 0.75

UC	XI	
0.00	0.10	○
0.00	0.20	△
0.00	0.30	+
0.00	0.40	x
0.05	0.48	◇
0.05	0.56	↑
0.05	0.66	x
0.05	0.76	z
0.05	0.86	z
	0.96	

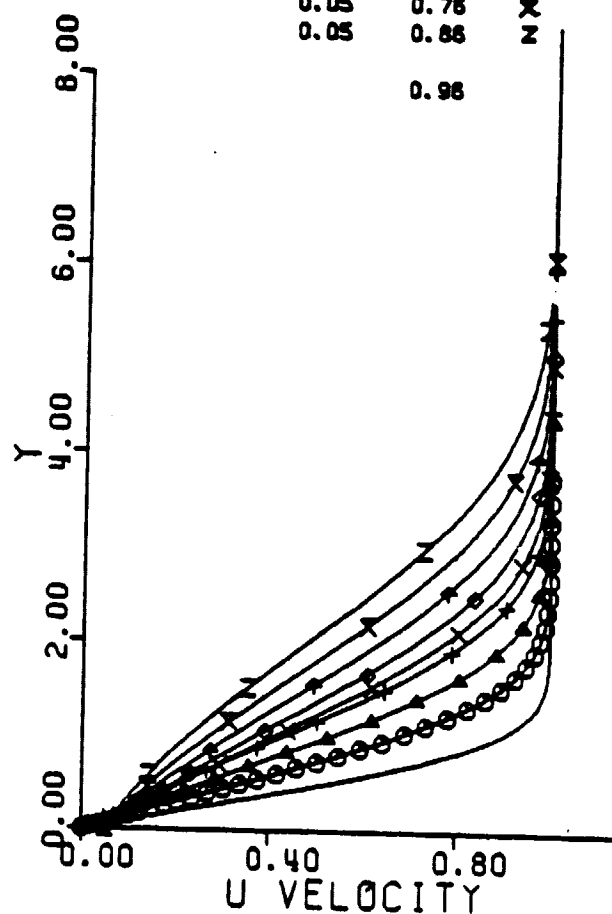
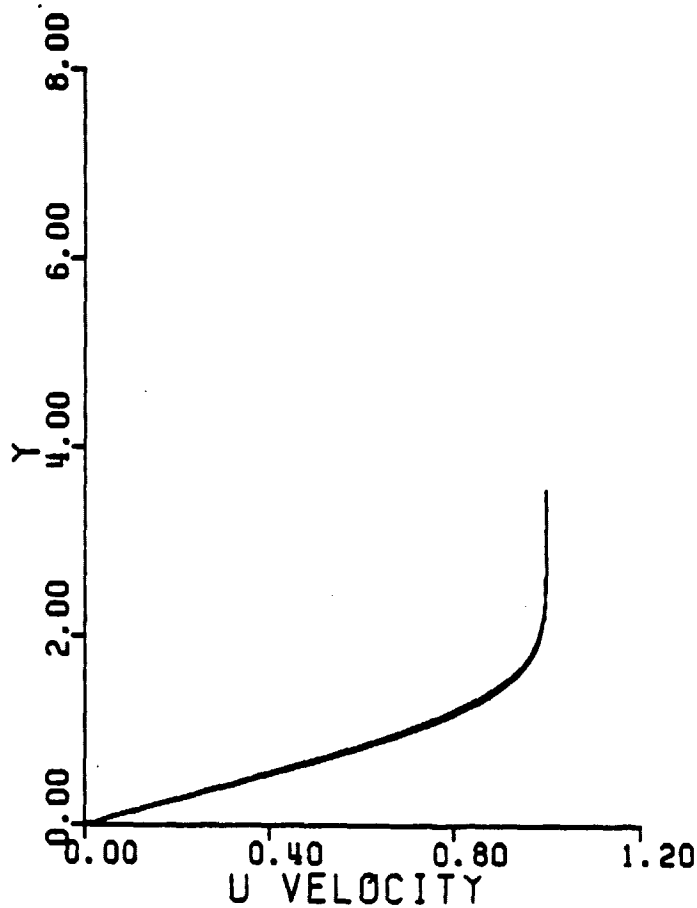


Fig. 1.8 The influence of blowing parallel to the wall;  
Time  $t = 0.75$

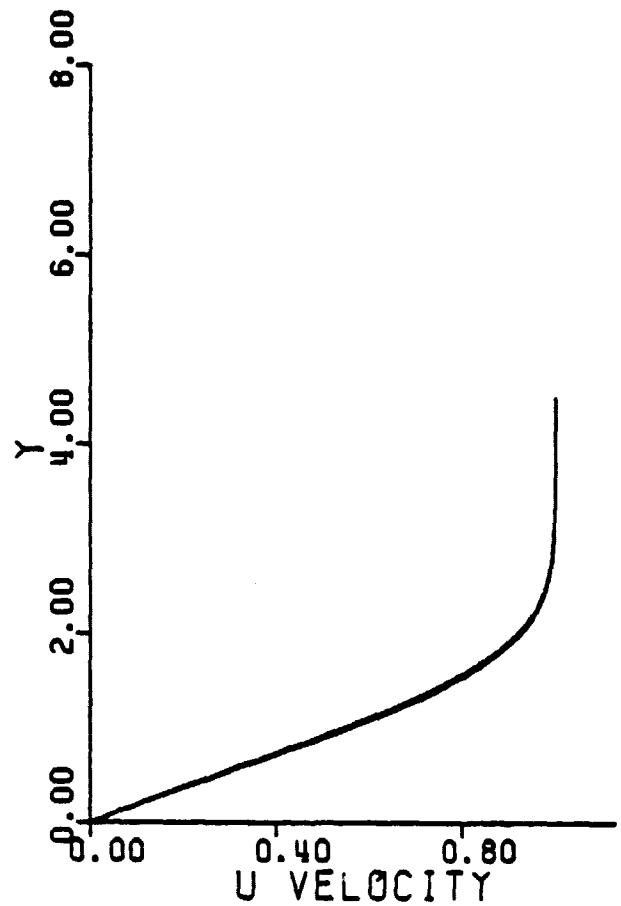


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XI  
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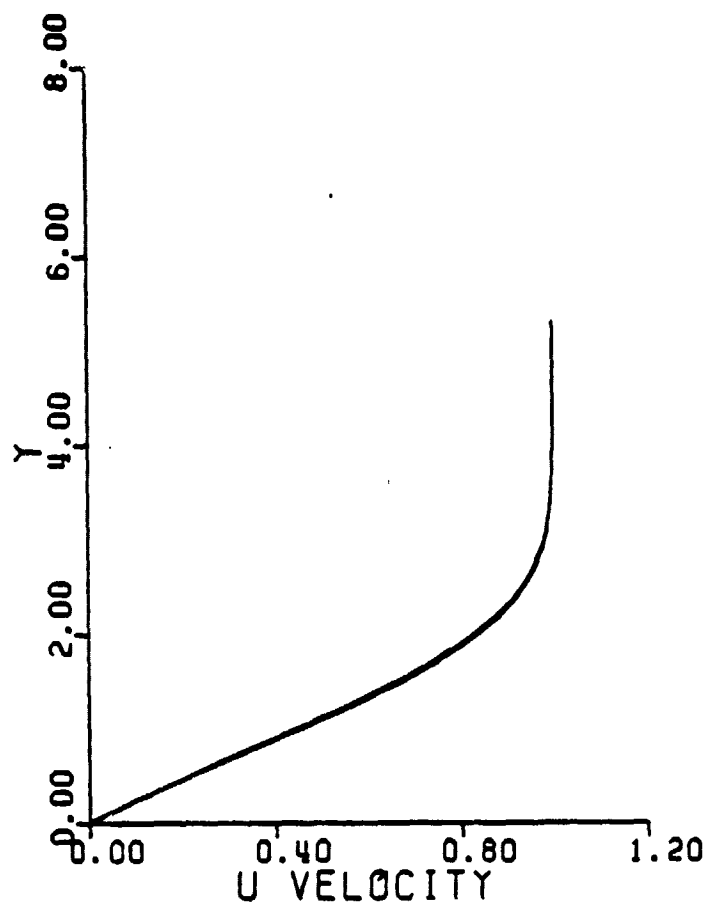
(b)

Fig. 1.9 Howarth flow with impulsive injection perpendicular to the wall  
for time  $t = 0, 0.25, 0.50, 0.75$ .

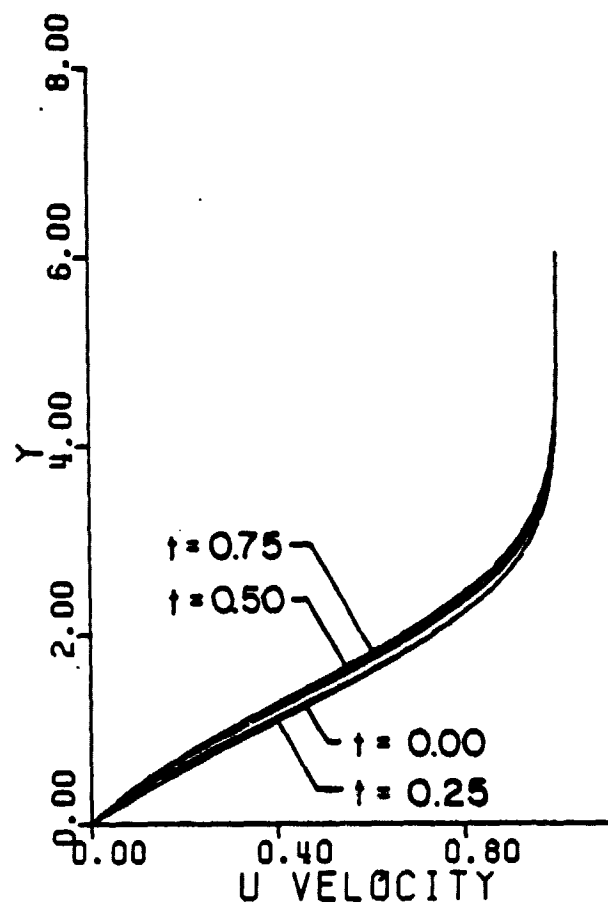
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(c)



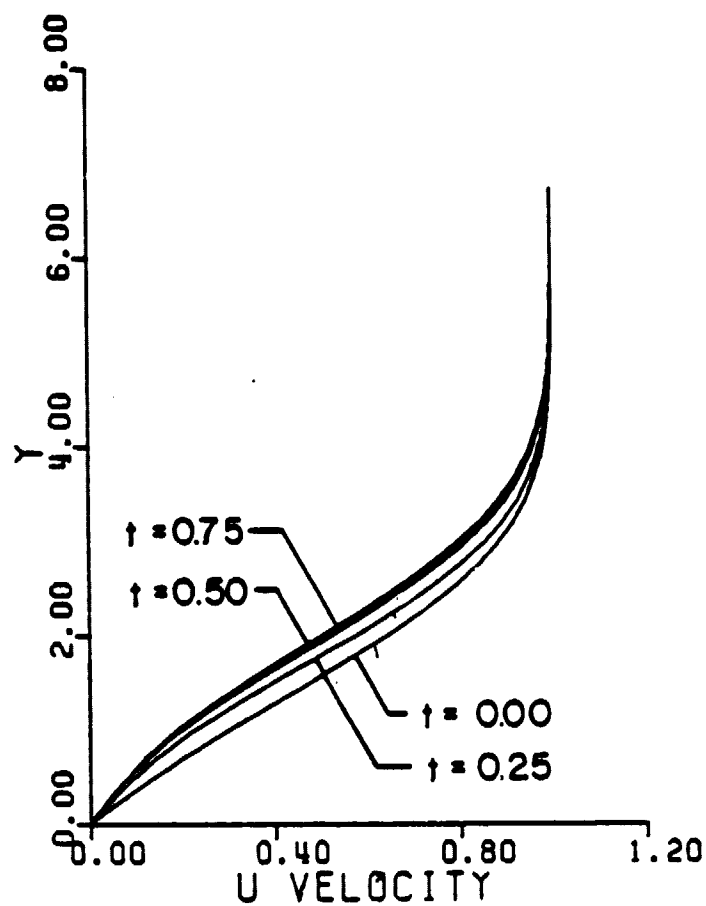
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Fig. 1.9 continued

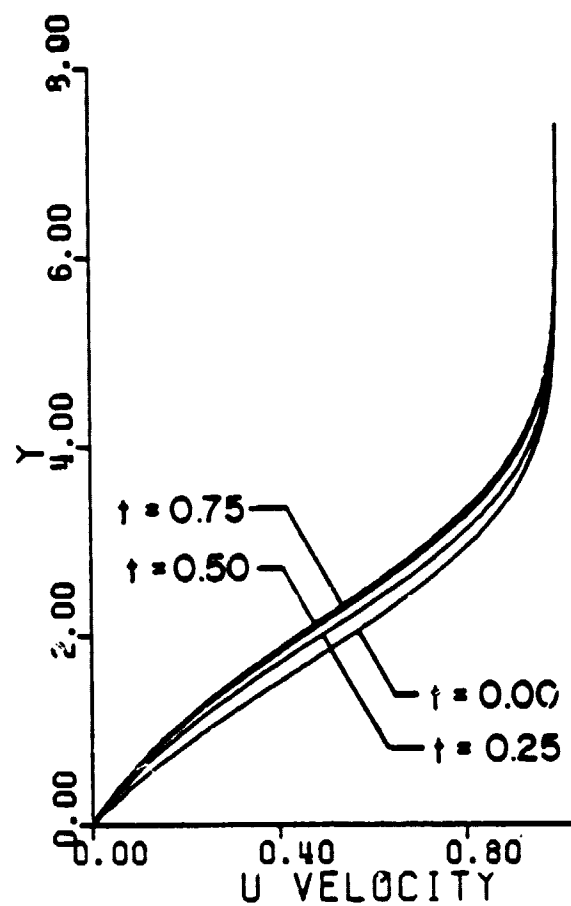
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(e)



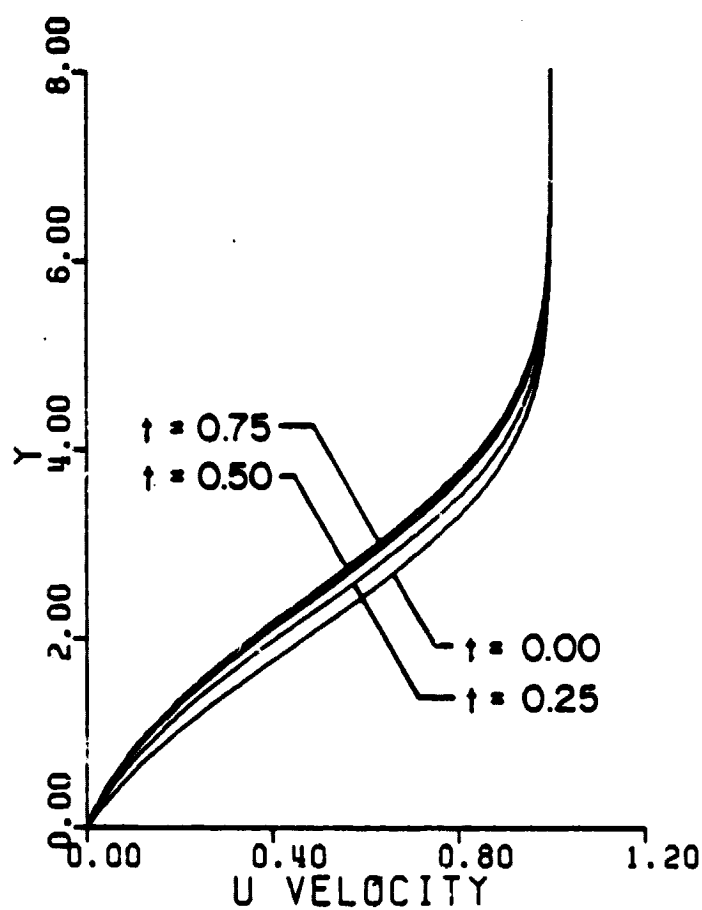
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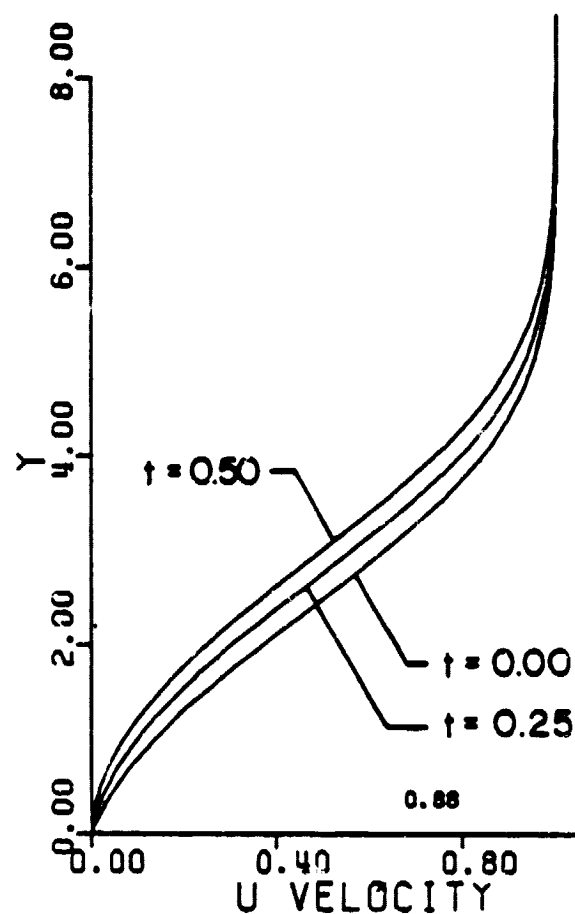
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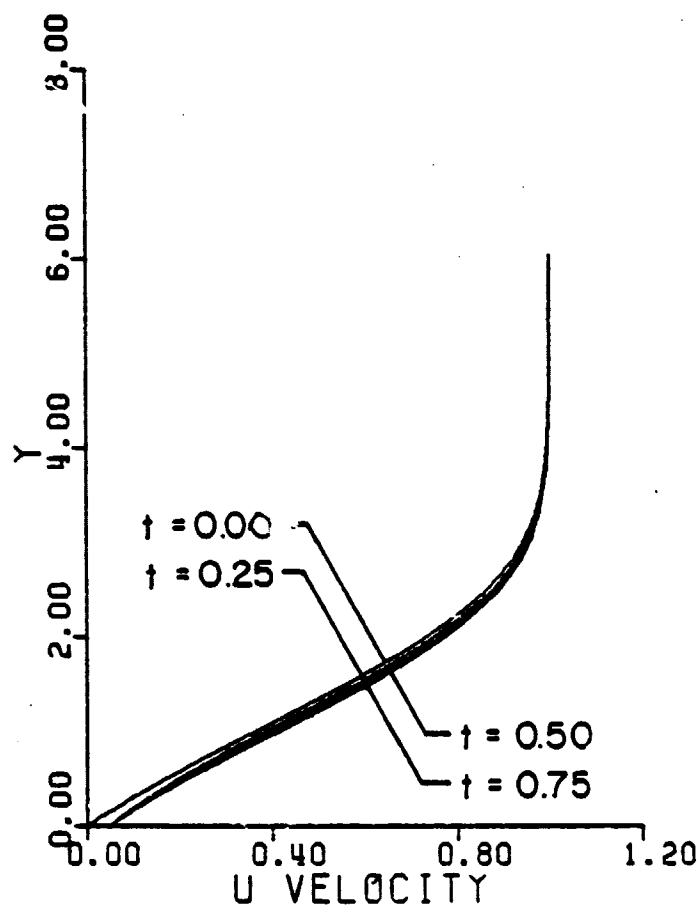


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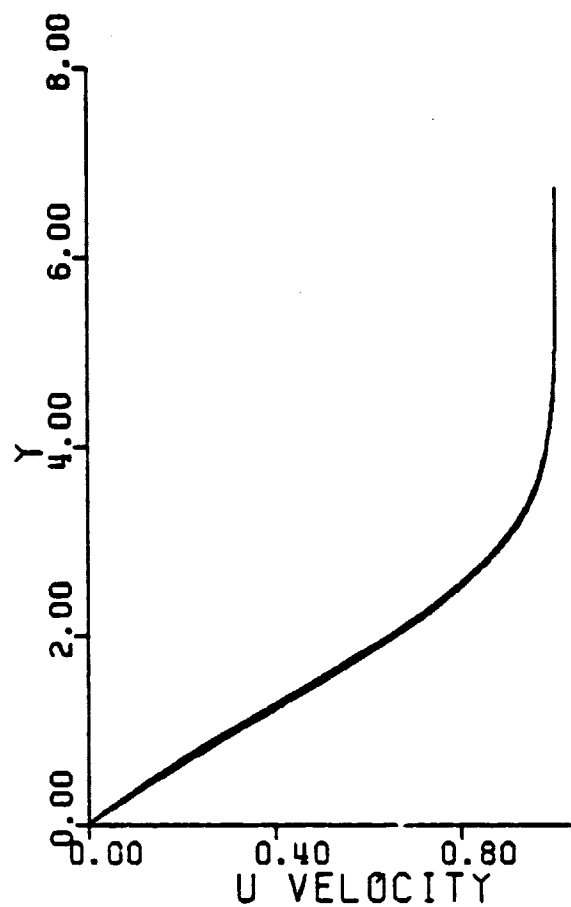


(h)

Fig. 1.9 continued

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(a)



(b)

Fig. 1.10 Howarth flow with unsteady injection parallel to the wall,  $u_w/U_e = 0.05$ , for times  $t = 0, 0.25, 0.50, 0.75$ .

XI  
0.48

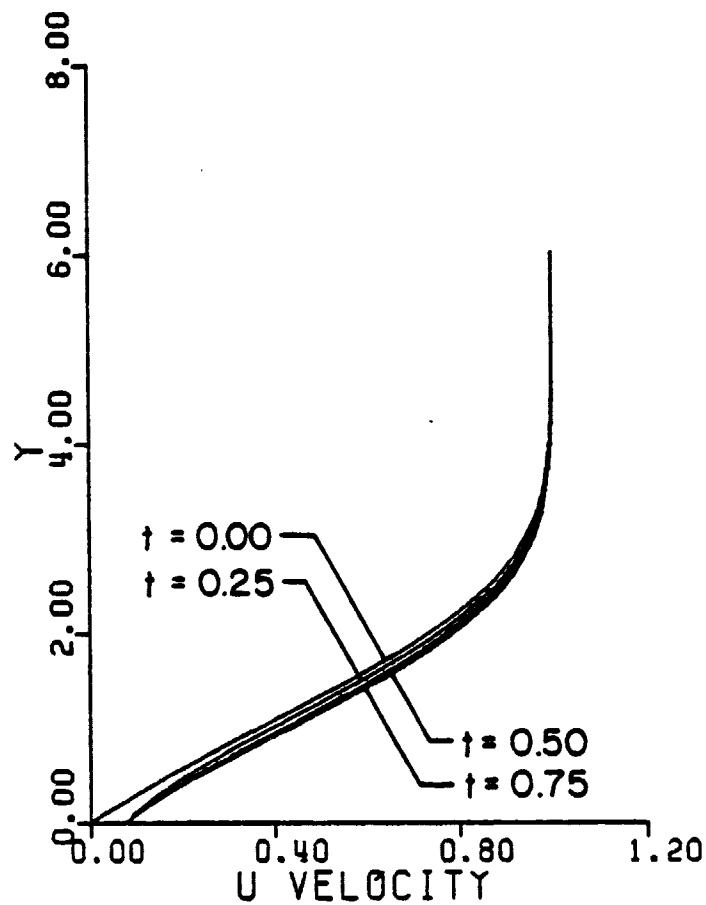
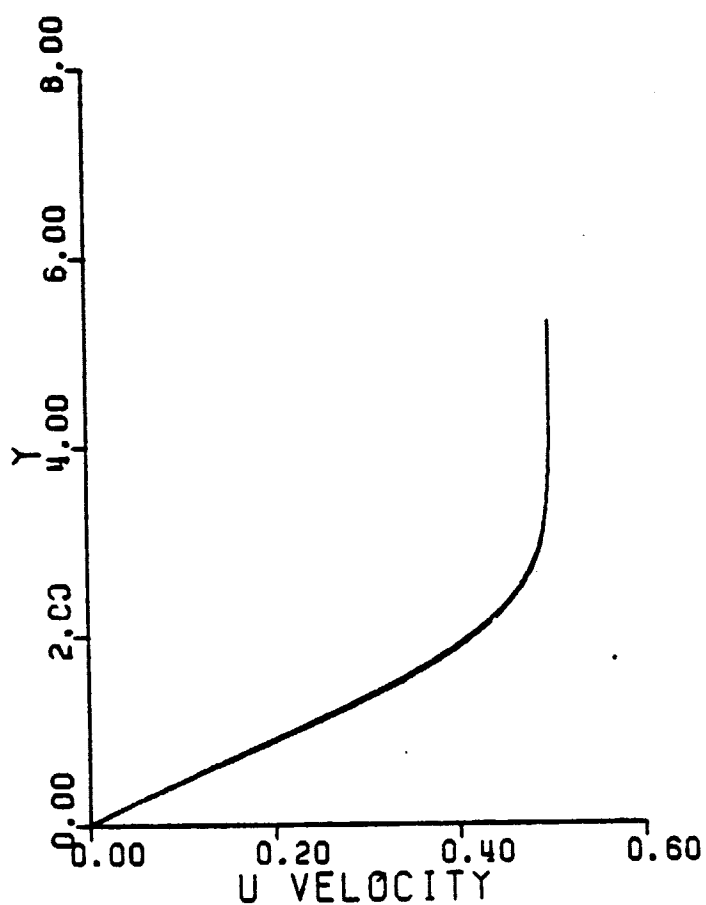


Fig. 1.11 Howarth flow with unsteady injection parallel to the wall,  $u_w/U_e = 0.075$  for time  $t = 0, 0.25, 0.50, 0.75$ .

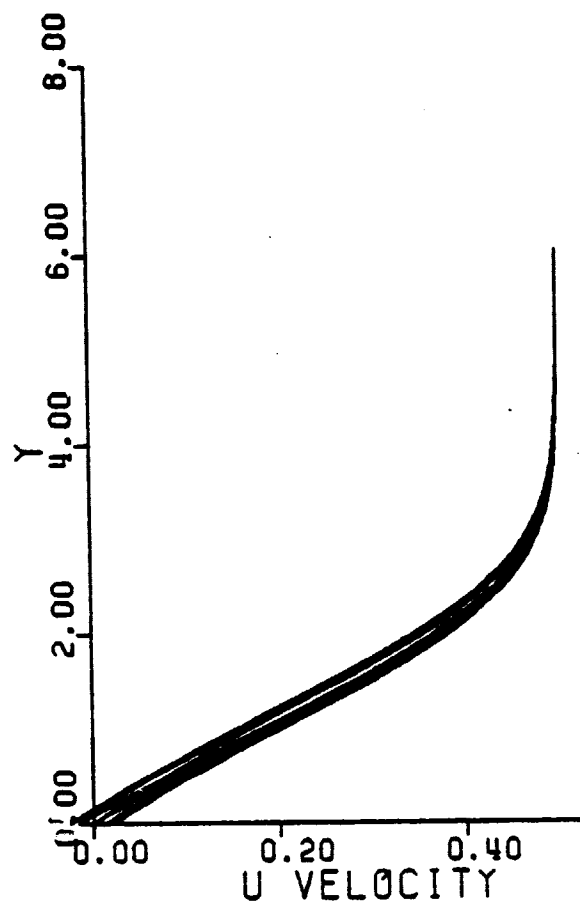
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0.48



(a)

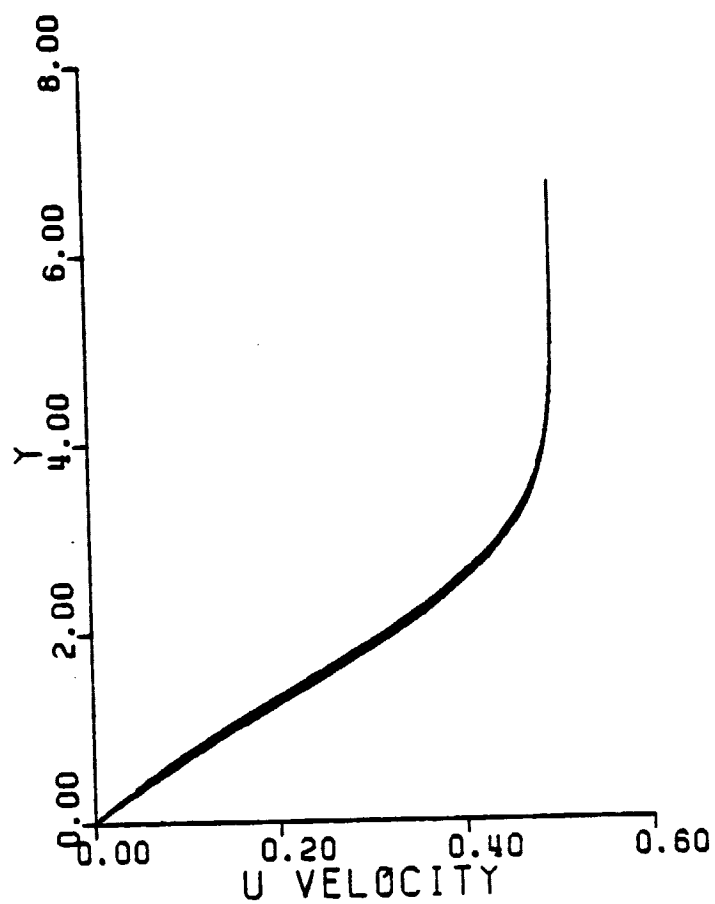


(b)

Fig. 1.12 Howarth flow with periodic injection parallel to the wall,  
 $u_w/u_e = 0.05$ .

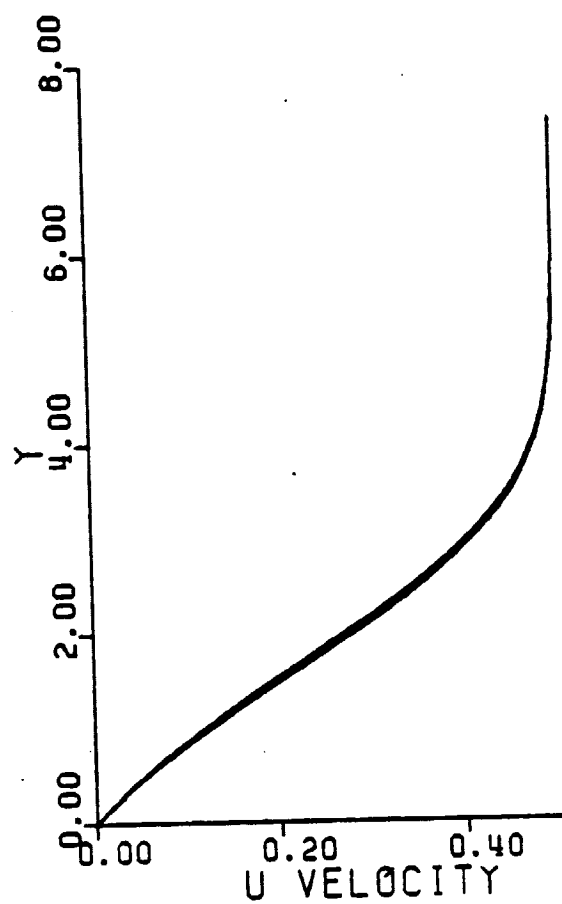
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OF POOR QUALITY

XI  
0.58



(c)

XI  
0.68



(d)

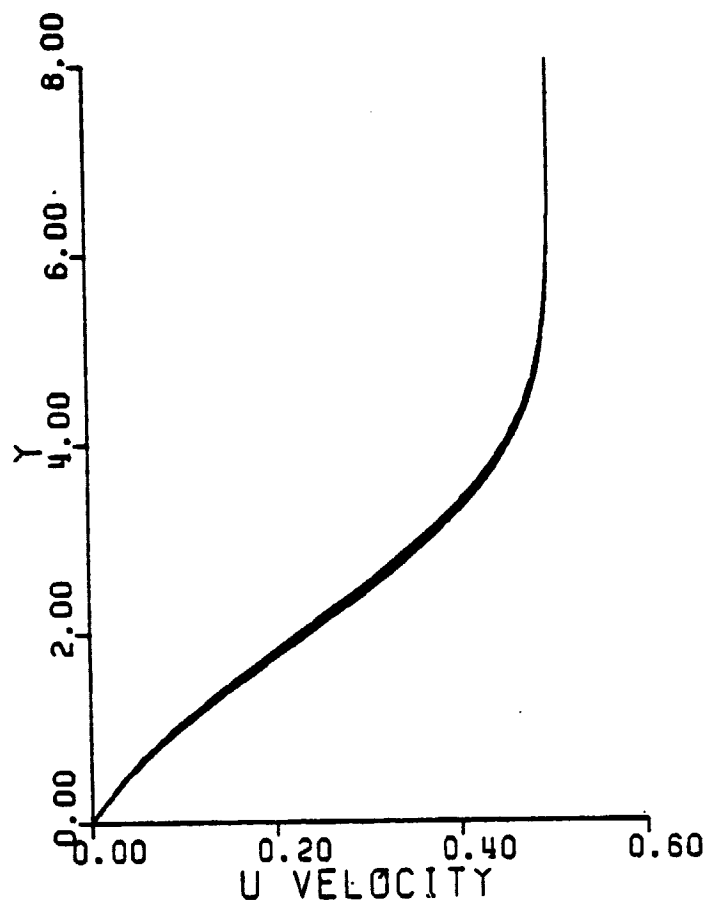
Fig. 1.12 continued



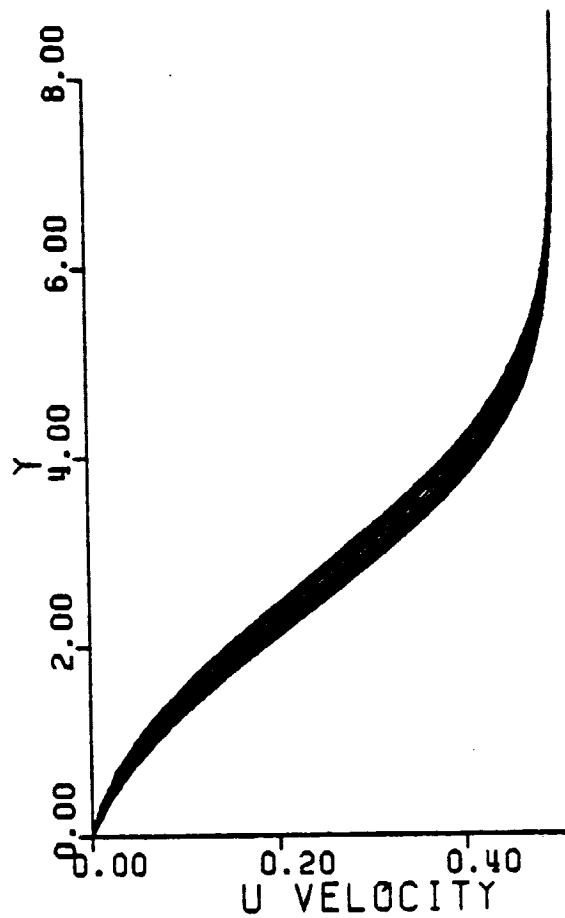
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XI  
0.78

XI  
0.88



(e)



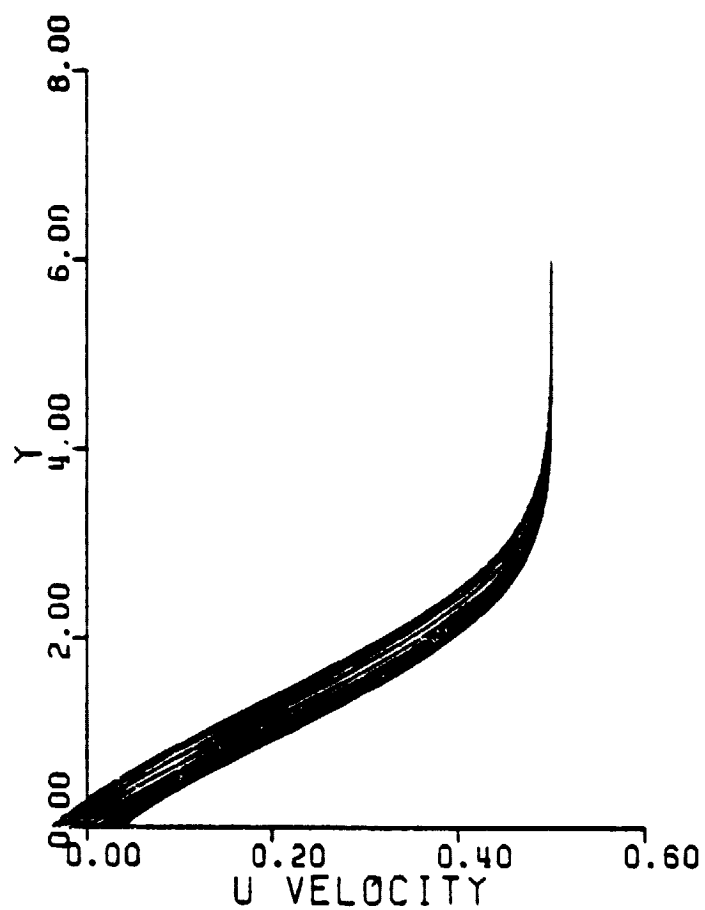
(f)

Fig. 1.12 continued

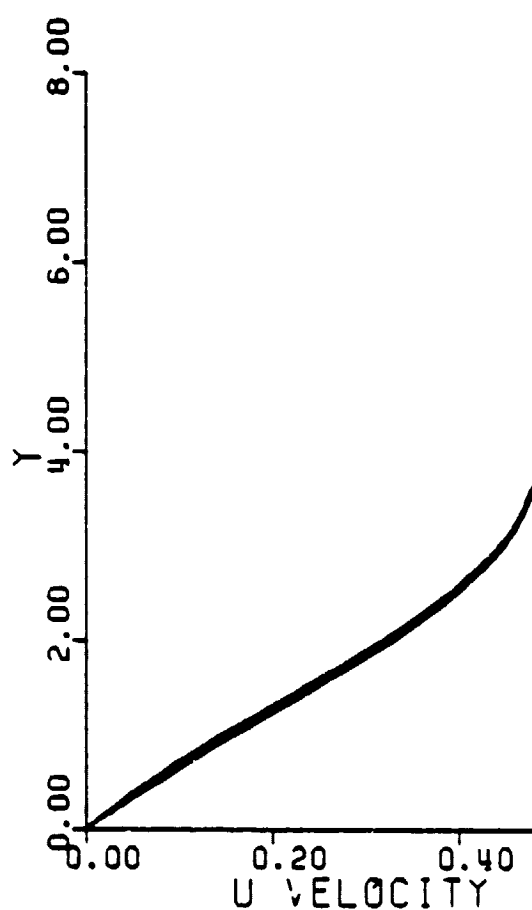
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OF POOR QUALITY

XI  
0.48

XI  
0.58



(a)

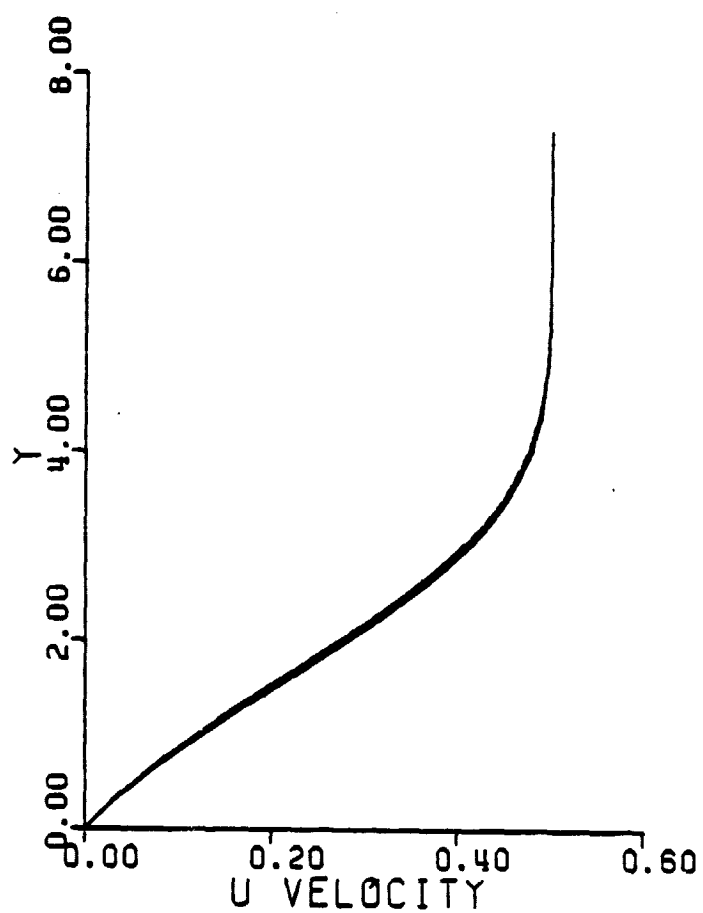


(b)

Fig. 1.13 Howarth flow with periodic injection parallel to the wall,  
 $u_w/u_e = 0.075$ .

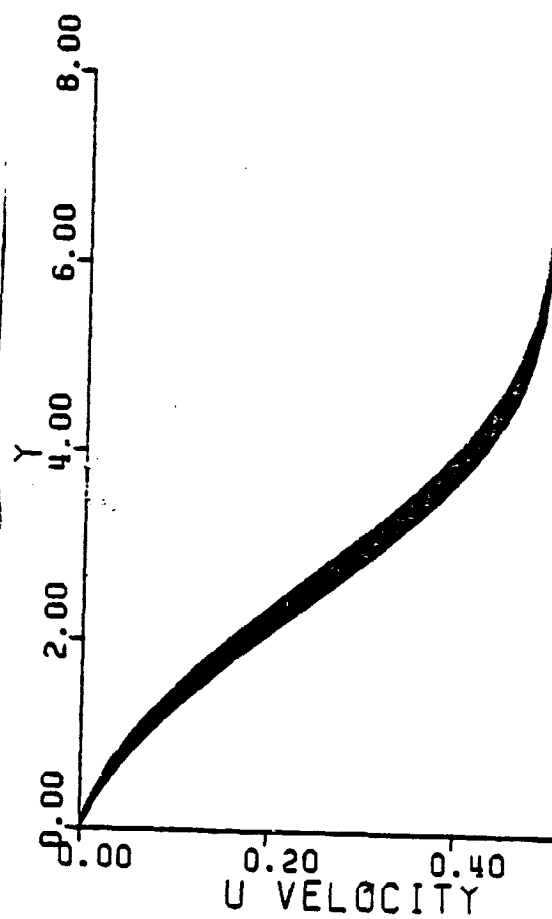
ORIGINAL PAGE IS  
OF POOR QUALITY

XI  
0.68



(c)

XI  
0.88



(d)

Fig. 1.13 continued

ORIGINAL PAGE IS  
OF POOR QUALITY

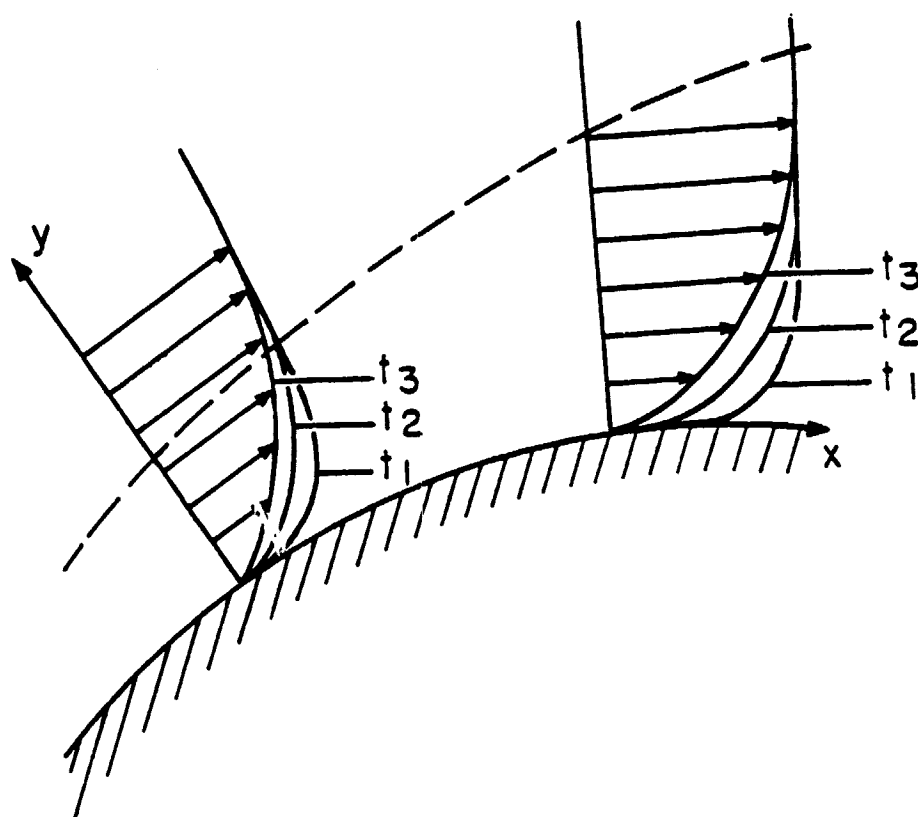


Fig. 2.1 Schematic profiles of an impulsively started boundary layer;  $t_1 < t_2 < t_3$

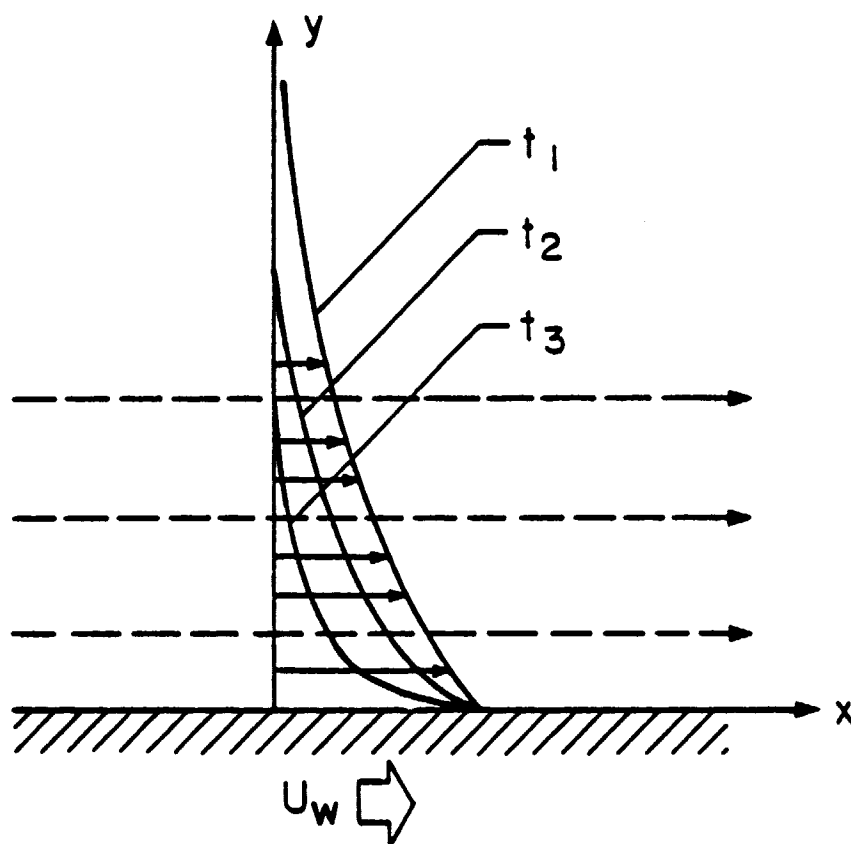


Fig. 2.2 Schematic profiles of an impulsively started flat plate in a fluid otherwise at rest;  $t_1 < t_2 < t_3$

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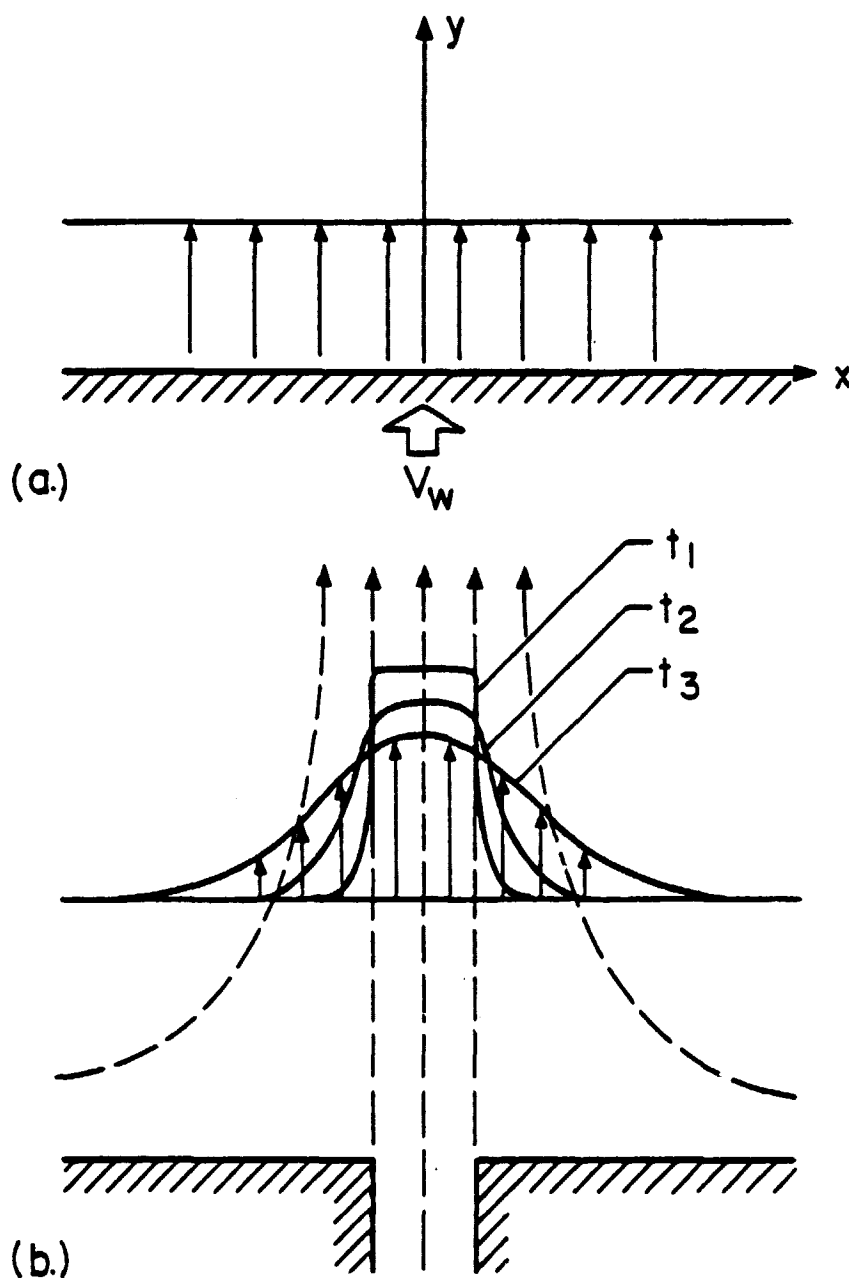


Fig. 2.3 (a) A flat plate started impulsively in the  $y$  direction  
(b) A jet started impulsively